ONE-DIMENSIONAL FLOW OF A COMPRESSIBLE GAS IN A PIPE IN THE PRESENCE OF A TRANSVERSE MAGNETIC FIELD

(ODNOMERNOE TECHENIE SZHIMAYEMOGO GAZA S KONECHNOI PROVODIMOST'IU PRI NALICHII POPERECHNOGO MAGNITNOGO POLIA)

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In [1] Golitsyn and Staniukovich have analysed the equations of one-dimensional flow of a compressible conducting gas in the presence of a transverse magnetic field.

Below we carry out the integration of their system of equations for the case of one-dimensional steady flow of a conducting perfect gas in the presence of a transverse magnetic field H. The viscosity and thermal conductivity of the gas are neglected. The axis of x is in the direction of flow of the gas. At the section $x = 0$ the parameters of state of the gas, its velocity and the strength of the magnetic field are given, i.e,

$$u = u_0, \quad p = p_0, \quad \rho = \rho_0, \quad T = T_0, \quad H = H_0 \text{ when } x = 0 \tag{1}$$

Let us assume that the currents flowing in the gas are connected to earth, so that spatial electric charges do not arise and electric fields are absent.

Let us take for our references of coordinate x, velocity of the gas u, pressure p, density ρ, temperature T and strength of magnetic field H, respectively, the quantities L, u_0, p_0, ρ_0, T_0 and H_0, where L is a characteristic dimension. In what follows we shall assume that all quantities are dimensionless without having recourse to a special notation.

Then the fundamental equations of the problem in dimensionless form are

$$\rho u \frac{du}{dx} + \frac{1}{kM_0^2}\frac{dp}{dx} = -SH\frac{dH}{dx}, \qquad \frac{d}{dx}\rho u = 0 \tag{2}$$

$$\frac{1}{k-1}\rho u\frac{dT}{dx} + p\frac{du}{dx} = \frac{SkM_0^2}{R_m}\left(\frac{dH}{dx}\right)^2, \quad p = \rho T, \quad \frac{dH}{dx} = R_m uH \tag{3}$$

where

$$M_0^2 = \frac{u_0^2}{kp_0/\rho_0}, \quad k = \frac{c_p}{c_v}, \quad S = \frac{\mu H_0^2}{\rho_0 u_0^2}, \quad R_m = \sigma\mu u_0 L \tag{4}$$

The boundary conditions of problem (1) in dimensionless terms have the form

$$u = p = \rho = T = H = 1 \text{ when } x = 0 \tag{5}$$

From Equations (2) we find two integrals of the system

$$u + \frac{1}{kM_0^2} p + S\frac{H^2}{2} = h = 1 + \frac{1}{kM_0^2} + \frac{S}{2}, \quad \rho u = 1 \tag{6}$$

Eliminating the temperature T from the first two equations of (3), and then the pressure p from the resulting equation by means of the first of Equations (2) and (6), we obtain the equation

$$\frac{k}{k-1}\left(h - \frac{k+1}{k}u\right)\frac{du}{dx} = \frac{k}{k-1}\frac{S}{2}H^2\frac{du}{dx} + \frac{S}{k-1}uH\frac{dH}{dx} + \frac{S}{R_m}\left(\frac{dH}{dx}\right)^2 \tag{7}$$

Integrating the third equation (3) with the boundary condition $H = 1$ when $x = 0$, we find an expression for the strength of the magnetic field in terms of the velocity of the gas

$$H = \exp\left(R_m \int_0^x u\,dx\right) \tag{8}$$

Substituting (8) in (7), we obtain the equation

$$\left(h - \frac{k+1}{k}u\right)\frac{du}{dx} = \frac{d}{dx}\left[\frac{S}{2}u\exp\left(2R_m\int_0^x u\,dx\right)\right] \tag{9}$$

Integrating (9) with the boundary condition $u = 1$ when $x = 0$, we have

$$\exp\left(2R_m\int_0^x u\,dx\right) = \frac{2}{Su}\left(hu - \frac{k+1}{2k}u^2 - h + \frac{k+1}{2k} + \frac{S}{2}\right) \tag{10}$$

Let us introduce the characteristic velocities

$$u_1 = \frac{kh + \sqrt{(kh-k-1)^2 + Sk(k+1)}}{k+1}, \qquad u_2 = \frac{kh - \sqrt{(kh-k-1)^2 + Sk(k+1)}}{k+1} \tag{11}$$

which are the roots of the equation

$$u^2 - \frac{2kh}{k+1}u + \frac{2kh}{k+1} - \frac{kS}{k+1} - 1 = 0 \tag{12}$$

Then Equation (10) has the form

$$\exp\left(2R_m\int_0^x u\,dx\right) = -\frac{(k+1)(u-u_1)(u-u_2)}{Sku} \tag{13}$$

Taking logarithms and then differentiating Equation (13), we obtain

$$2R_m\,dx = \frac{u\,(u-u_1) + u\,(u-u_2) - (u-u_1)\,(u-u_2)}{u^2\,(u-u_1)\,(u-u_2)}\,du \qquad (14)$$

Integrating (14) and making use of the boundary condition for the velocity, we find that

$$2R_m\,x = \frac{1}{u_1}\ln\frac{u-u_1}{u\,(1-u_1)} + \frac{1}{u_2}\ln\frac{u-u_2}{u\,(1-u_2)} + \frac{1-u}{u} \qquad (15)$$

Equation (14) can be rewritten in the form

$$2R_m\,dx = \frac{u^2-u_1u_2}{u^2\,(u-u_1)\,(u-u_2)}\,du \qquad \left(u_1u_2 = \frac{1+{}^1/_2\,(k-1)\,M_0^2}{{}^1/_2\,(k+1)\,M_0^2}\right) \qquad (16)$$

When $M_0 > 1$ we obtain a continuous deceleration of the stream from the velocity $u = u_1$ when $x = -\infty$ to the velocity $u = \sqrt{u_1u_2}$ (local Mach number $M = 1$). When $M_0 < 1$ the stream continuously accelerates from the velocity $u = u_2$ when $x = -\infty$ up to the velocity $u = \sqrt{u_1u_2}$.

For the strength of the magnetic field H we have the formula

$$H = \sqrt{-\frac{(k+1)\,(u-u_1)\,(u-u_2)}{Sku}} \qquad (17)$$

The pressure p, density ρ and temperature T are found respectively from Equation (6) and the second equation (3).

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